Estimating the reliability of the primary support for a given tunnel section

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ABSTRACT: Uncertainty dominates design in rock engineering to such an extent that it sometimes raises doubts about the realism of a design solution. Using Monte Carlo simulation and the ground-support interaction model different sources of uncertainty can be integrated into the analysis allowing for determination of the reliability of a support section. Two example applications are presented which aim at demonstrating the benefits from adopting the proposed technique both as a back-analysis and a design tool for tunnel design. The binomial distribution is used to assist in the definition of the acceptable level of probability of failure which is a function of the application length of a section type. The role of correlation between variables defining the support capacity and the required pressure for ground-support equilibrium is discussed with reference to design variability (considered manageable at the face) and uncertainty (considered as unmanageable). Despite some limitations the proposed approach provides a meaningful and practical technique for evaluating the reliability of a tunnel support section.

1 INTRODUCTION

The analysis of ground-support systems is conducted with the assumption that the behavior of the ground and that of the support system are well-understood and quantifiable. In general, the resulting design solution does not consider uncertainty in the ground-structure interaction, variability in construction material properties, and variation in the geometry of the structure during construction.

Uncertainty associated with ground parameters arises from the inherent variability of the ground, inability to test the actual behavior of the ground in-situ, biases in sampling, testing and processing of the results. Uncertainty is also associated with the modeling procedure itself and the simplifications made during the process of analysis.

Although decisions under uncertainty are routinely dealt with in other fields of engineering and science, they are only occasionally addressed in tunneling due to the complexity of the problems and a lack of suitable and effective tools. In fact, the current, common tunnel design practice, particularly the dimensioning of primary support and final lining, is to use deterministic approaches incorporating engineering judgment and established design principles.

The so-called “best-estimates” cannot account for either the inherent variability or the uncertainty in the parameters, and the factor of safety, commonly defined as the ratio between the available capacity of the designed support and the demand for support of the excavation, is often found to be inadequate for quantifying the reliability of the system. As shown in Table 1 different tunnel sections having the same factor of safety may have a quite different probability of failure (Pf), depending not only on the variability of the demand and capacity of the system, but also on the correlation between these two measures.

In the following sections, basic elements of reliability-based design are briefly presented, and a simple, yet practical, technique is proposed that aims at allowing the designer to consider uncertainty in a ground-support system in a meaningful way. Example applications for railway tunnels are presented. A discussion on induced correlations amongst geomechanical and construction variables and on the meaning of Pf follows.

2 PRINCIPLES OF RELIABILITY-BASED DESIGN

A probabilistic study allows uncertainty related to a parameter (or a random variable) to be integrated in the analysis through the use of probability density functions (pdfs). Various sources of uncertainty can be compared, analyzed and combined using a probabilistic procedure. For a given level of uncertainty in the problem, the implied level of reliability can also be quantified, thus allowing comparison of the safety (reliability) of alternative designs (Tang 1993).

Table 1. Probability of failure for factor of safety equal to 1.3, and different combinations of demand and capacity variability, and correlation between capacity and demand.

CoVD	CoVC/CoVD	$\rho_{C,D}$	Pf
0.250	0.750	0.250	0.095
		0.500	0.063
		0.750	0.030
	1.000	0.250	0.105
		0.500	0.063
		0.750	0.020
0.400	0.750	0.250	0.229
		0.500	0.178
		0.750	0.113
	1.000	0.250	0.244
		0.500	0.178
		0.750	0.087

CoV: coefficient of variation, C: capacity, D: demand, $\rho_{C,D}$: correlation coefficient between C and D, and Pf: probability of failure.

Traditionally, engineering assessment of failure risks is made through comparison of the calculated central factor of safety (CFS), defined as the expected capacity ($\overline{C}$) divided by the expected demand ($\overline{D}$) for the system under consideration, with the "allowable FS" established from previous experiences. This index, although practical, does not depict the variability of the system parameters (Harr 1987).

A more effective way to assess the reliability of a design solution is to consider the safety margin (S), which is defined by the difference between capacity (C) and demand (D). Inadequacy of a design is considered within the negative portion of the safety margin distribution:

$$Pf = P[(C-D)\leq 0] = P[S\leq 0] \tag{2.1}$$

Another measure of a design adequacy is the reliability index, β, defined as the arithmetic average of S, $\overline{S}$, over the standard deviation of S, $\sigma(S)$:

$$\beta=\overline{S}/\sigma(S) \tag{2.2}$$

In general, any reliability-based analyses shall consist of the following steps:

1. Definition of an empirical, analytical or numerical model that is more suitable for the ground conditions - structure interface. This is important, because there is often the misconception that reliability-based analysis can protect the designer from errors arising from selecting an inappropriate modeling technique for the system under investigation.
2. Definition of the character of the input variables, deterministic or probabilistic (stochastic).
3. Fitting of the appropriate pdf to the observed data and/or assignment of an adequate pdf to the stochastic variables. Sometimes, this process can be aided by the principal of maximum entropy (Harr 1987), or information available in literature regarding the coefficient of variation of typical design variables. In most cases, due to shortage of data, the selected pdf corresponds to either a normal or a triangular distribution. Nevertheless, when sufficient data are available (at least 20 measurements), standard procedures can be used for fitting a pdf to this data.
4. Incorporation of the different sources of uncertainty in the design analysis methods. There are mainly three approaches for doing so:

- Monte Carlo (MC) simulation (Metropolis & Ulam 1949) where repeated samples are taken from actual or estimated pdf of the variables which enter in a function Φ (e.g. support capacity) until the distribution of this function is defined with acceptable precision.
- Taylor series (First Order Second Moment, FOSM method), where Taylor's formula is used for expanding a function Φ about the average value $\bar{x}$ up to the quadratic term. The expected value and variance are then calculated. The application of this technique is difficult for multivariate functions, even impossible for the cases for where the function is not given in an analytical form (e.g. numerical models).
- The Point Estimate Method, PEM (Rosenblueth 1975), where only two values for each input variable are used to calculate the basic moments of a function. The point estimates of Φ are subsequently multiplied by the function values to accordingly define its central moments. The authors have successfully combined PEM with boundary and finite element codes for tunnel design (Kalamaras 1996, Russo et al. 1999).

5. Application of the probabilistic technique to define the reliability of a design solution, and investigation of its sensitivity to the input variates values, including inherent and induced correlations amongst themselves. Comparative evaluation between different design solutions with reference to norms and previous experiences to select the design solution which poses the least uncertainty for the geologic conditions expected.

6. Optimization of the construction practice to maximize the reliability of the design solution selected.

The last two aspects are further discussed in section 5.

3 EVALUATION OF THE RELIABILITY OF A PRIMARY SUPPORT SYSTEM

The convergence-confinement technique is a well-established aid to the design of tunnel support systems. Modeling of the ground behavior is based on a mathematical solution to the problem of progressive formation of a failure zone around the tunnel. It is the interaction between this progressive failure and the response of the support system that provides the basis for calculating the support that has to be installed to provide stable tunneling conditions.

The ground-support system is considered to be in equilibrium when the support line intersects the rock mass displacement curve before unacceptable displacement has occurred. At that point, the pressure exerted on the support system is the pressure required to attain equilibrium conditions, p_{ireq}. The support line is defined by its start at u_{so} (deformation of the excavation at the time of support installation), and a slope equal to the stiffness of the support, k_S, normalized by the equivalent tunnel radius, r_i.

In the case that the available support capacity, p_s, is inadequate, that is $p_s<p_{ireq}$, demand is considered to have surpassed capacity resulting in possible yield of the support system before reaching the equilibrium condition. In the case of an equilibrium condition, that is $p_s \geq p_{ireq}$, a verification of the deformation value and the extent of the plastic zone at equilibrium are necessary to check if these two measures are within pre-defined allowable limits (e.g. 2%, and $2r_i$.). If either of these two limits is exceeded, i.e. the available support stiffness (k_S) is insufficient, the demand for stiffness (k_D) is considered to have surpassed the available capacity ($k_S<k_D$).

Both cases of $p_s<p_{ireq}$ and $k_S<k_D$ can be readily identified during the analysis process; however, in this paper they are treated together as the same case of support inadequacy and in terms of the capacity-demand model simply as C<D.

The ground-support interaction model adopted includes the following basic elements:

- the closed-form solution of Brown et al. (1983), for elastic-brittle-plastic material is used to model the response of the ground (further developments include a strain softening model, and the incorporation of the concept of rock mass requalification according to Mahtab et al. 1994);
- a part or the entirety of the gravitational load of the plastic zone on the crown (Hoek & Brown 1980) can be taken into account in the analysis;
- the Hoek-Brown failure criterion parameters, *m* and *s*, are derived from the Geological Strength Index (*GSI*, Hoek 1994 and Hoek & Brown 1997); the *m* and *s* values corresponding to residual conditions are functions of a reduced *GSI* value (GSI_{RES}, Russo et al. 1998);
- as for the dilatancy parameter, the average of the values corresponding to the hypothesis of deformation at constant volume and to the maximum dilatancy calculated according to the theory of plasticity (i.e. associated flow rule), is used; [the possibility to relate rock mass conditions with dilatancy coefficient is currently investigated];
- the support capacity and stiffness are first calculated according to the equations proposed by Hoek & Brown (1980) and then reduced accordingly by the so-called shape factors, which account for the non-circular excavation cross-section and are derived from parametric numerical analysis;
- the critical parameter of deformation at the time of support installation (u_{so}) is calculated according to the principle of similitude (Nguyen Minh & Corbetta 1992) at a distance *d* from the face, which is a function of the rock mass conditions and construction practice (e.g.: for $GSI \leq 35$ $d=0.2\times r_i$, for $35<GSI \leq 75$ $d=0.8\times r_i$ and for $GSI>75$ $d=1.5\times r_i$ or d=2×steel set spacing);
- random numbers are generated according to the Latin hypercube sampling scheme due to its superiority over simple random sampling in yielding more precise results in fewer iterations (Iman and Helton, 1985); intrinsic and induced correlations between rock mass properties and between rock mass quality indices and construction parameters, respectively, are introduced in the Monte Carlo simulation using rank correlation. Demand, p_{ireq}, is defined using the bisection method.

The analysis procedure is shown schematically in Figure 1.

4 EXAMPLE APPLICATIONS

4.1 *Application 1: Back-analysis of observed support performance*

This first application refers to the “back-analysis” of the behavior of a ground-support system for the top-heading of a headrace tunnel in India. An extrapolation of the behavior of the support system for the next phase of enlargement of the tunnel to its final size had also to be made. The excavation of the 10m-diameter tunnel through densely-jointed, quartz-mica schists of low strength under an overburden ranging between 600 and 800m resulted in large convergence. Cracks on the shotcrete and bending of the steel ribs were observed along approximately 50% of the alignment (about 1000m).

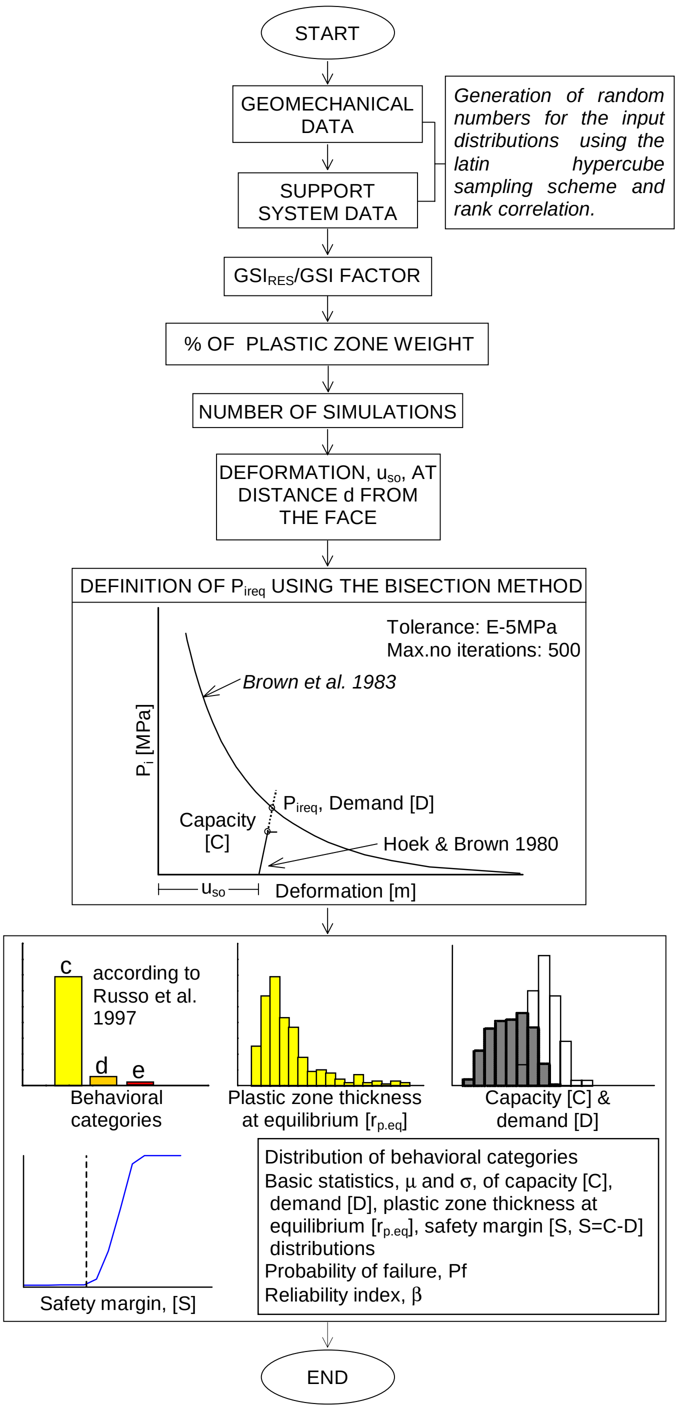


Figure 1. Scheme for reliability evaluation of a primary support system using Monte Carlo simulation and the convergence-confinement method.

The data recorded during the excavation allowed for the definition of the distributions given in Table 2. A significant correlation (ρ=0.85) was found between GSI and the uniaxial compressive strength of the rock. According to the recorded data, a correlation between geomechanical and construction parameters was also included in the analysis, to take into account that the density of the support system was regulated in function of the observed rock mass conditions at the face.

Table 2. Statistical distributions and correlations for the geological and construction variables used in Applications 1 and 2.

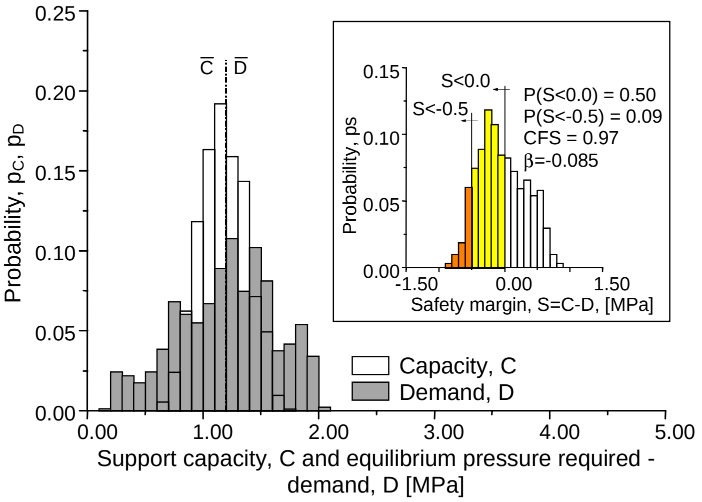
Input variables	Application 1	Application 2
Intact rock strength, σ_c, MPa	custom pdf of bimodal type, range: [14, 85]	[10,20,50]*
Geological Strength Index, *GSI*	fitted lognormal distribution (μ = 40, σ = 7)	[10,20,50]
Shotcrete thickness, $t_{shot.}$, m	[0.10,0.20,0.25]	original: [0.05,0.10,0.15], alternative: [0.15,0.20,0.25]
Shotcrete strength, $\sigma_{c.shot.}$, MPa	[10,12,15]	[10,12,15]
Steel set spacing, *S*, m	[0.75,1.00,1.50]	alternative: [1.00,1.25,1.50]
Longitudinal & circumferential bolt spacing, s_c & s_l, m	[1.35,1.50,1.65]	original: [0.90,1.00,1.10]
Bolt capacity, T_{bf}, MN	[0.10,0.15,0.20]	original: [0.10,0.15,0.20]

Correlation matrix

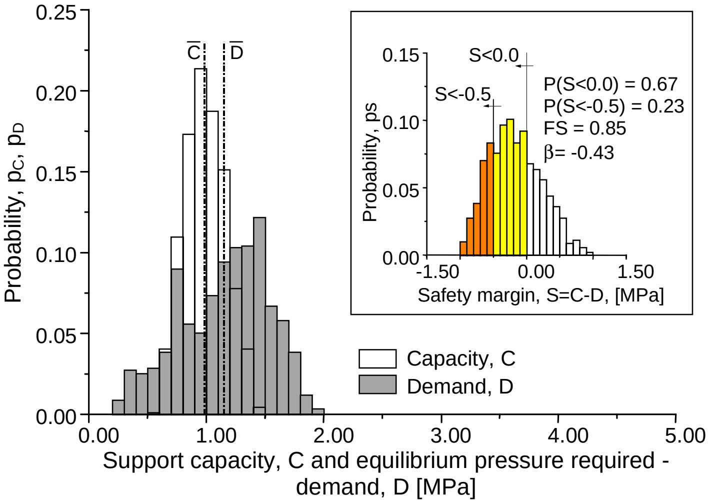
	σ_c	*GSI*	$t_{shot.}$	s_c & s_l	*S*
σ_c		0.85**	-0.50	0.50	0.50
GSI	0.85**		-0.75	0.75	0.75
$t_{shot.}$	-0.50	-0.75		-0.75	-0.75
s_c & s_l	0.50	0.75	-0.75		-0.75
S	0.50	0.75	-0.75	-0.75	

*Unless specified, pdfs are otherwise of triangular type and defined by [min,mode,max].
**The only existing correlation, found in the recorded data, is between σ_c and *GSI* for Application 1 only. All others are induced correlations.

The simulation results are presented in Figure 2a. The main figure provides the distributions of support capacity, C, and pressure required for equilibrium, D, while the inclusion figure the distribution of the safety margin, S. It is worth stating here that the probability of failure is not represented by the intersection area under the C and D distributions, because these two variables are actually correlated as a result of the correlations induced between the geomechanical variables and the construction indices.



(a)



(b)

Figure 2. Distributions of support capacity, pressure required for equilibrium, and safety margin, for top-heading (a) and for enlargement (b), of a headrace tunnel in India.

It can be said that modeling of the site conditions with the proposed approach is successful since in 50% of the cases demand is greater than capacity, a situation which was also observed on site. In fact, for approximately 7% of the alignment, severe steel-set bending and intense cracking in the shotcrete were observed. For practical purposes, this percentage corresponds to S<-0.5MPa.

Assuming that values of S below 0.0 and -0.5MPa represent respectively light and heavy support inadequacy, the enlargement of the excavation could result in instabilities for 67% of the alignment, out of which 23% could be characterized as intense (Figure 2b). Considering that the installed support at the top heading had already undergone significant deformation at equilibrium conditions, a general inadequacy of the support for the enlarged section was foreseen. Redesign of the support system followed, using numerical modeling and back-analysis of the observed behavior.

4.2 *Application 2: Optimization of a primary support system*

The second application refers to the design of a 100m^2 railway tunnel in South America, excavated in complex metamorphic rocks exhibiting significant schistocity. The original design included shotcrete and bolts as the primary support system for fair rock mass conditions and for an overburden ranging between 150 to 200m. Amongst other technical and economical considerations, the effectiveness of the original support system for the range of anticipated geomechanical conditions and variability of construction parameters had to be established.

The procedure in Figure 1 was employed using the parameters distributions given in Table 2. As it is shown in Figure 3, the analysis revealed that the original design solution was associated with an unacceptable probability of failure, Pf (19%), despite that the CFS value was equal to approximately 1.2 . The cumulative, safety margin distribution for the proposed alternative solution, consisting of steel ribs and shotcrete, is also given in Figure 3, from which it can be seen that this design solution has only a Pf of less than 1%. This alternative solution was finally selected since, apart from its design superiority, it was also associated with favorable cost and advance rate indices.

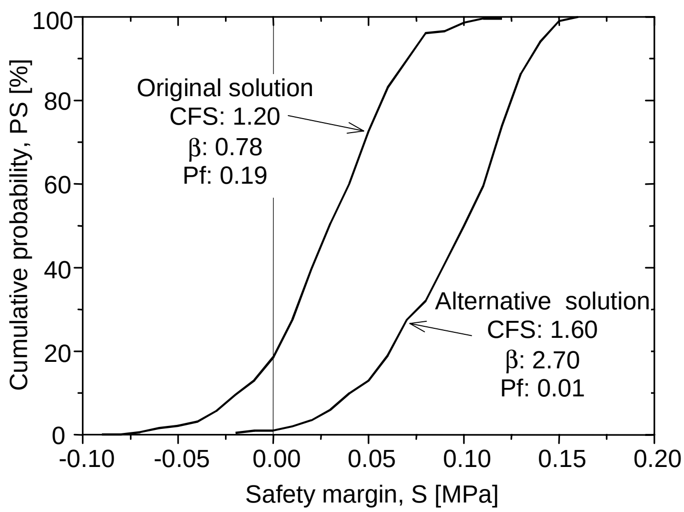


Figure 3. Cumulative distributions of safety margin for original (shotcrete and rockbolts) and alternative (shotcrete and steel sets) primary support solutions for a railway tunnel in South America.

5 SELECTING THE ACCEPTABLE LEVEL OF PROBABILITY OF FAILURE AND MANAGING VARIABILITY DURING EXCAVATION

Moving from deterministic to reliability-based design in rock engineering requires the specification of an acceptable level of Pf that a design solution has to satisfy. Although information is provided in literature on threshold values for earthworks (e.g. Carter 1992) and temporary mine openings (e.g. McCracken & Stacey 1989), no specific values for the required reliability of a support system of a civil tunnel is specified to the authors' knowledge. An insight into this problem can be gained by considering that the Pf calculated with the proposed method corresponds to the two-dimensional (2D) probability of failure each time the support section is applied, rather than the 3D failure along the tunnel where this section is applied. The latter can be calculated using a binomial distribution, according to which the probability of at least one failure, PF, is:

$$\mathrm{P_F}=1-[(1-\mathrm{Pf})^{n}] \qquad (5.1)$$

where n is the number of Bernoulli trials which corresponds to the total number of times the section type is applied along the tunnel. In the analysis, n, is calculated as the ratio between the accumulative, tunnel segment length of application of the section type and the designed section length which is equal to the average spacing of the section type. The procedure is schematically presented in Figure 4.

Alternatively, the Poisson distribution can be used, when n is large and the statistical distribution of the expected number of failures is of interest such as in the case of risk analysis. Apparently, the acceptable level of Pf is a function of the length of the tunnel where the section type is to be applied. As Figure 5 indicates, assuming the same acceptable level of PF, higher Pf can be acceptable for segments of limited extent, while lower for sections applied over longer tunnel segments.

In defining the input values for the structural reliability analysis, two types of uncertainty are identified: Type 1 which represents the variability (intrinsic or induced) of a parameter and Type 2 which is related to the uncertainty and subjectivity in evaluating the actual state of a parameter. For example, for a certain tunnel section it is possible to consider for a specified design value of shotcrete thickness of 15cm a Type 1 design variation of ±7cm, to be defined at the tunnel face with reference to the geomechanical conditions. In addition, an unmanageable, Type 2 variation of ±3cm can also be introduced to reflect the practical operational difficulties in assuring the designed shotcrete thickness.

This concept is of particular importance because it permits to differentiate in the design analysis between the variation in the stabilization measures to be managed during construction, and the uncertainty (Type 2) which cannot be managed but only considered in the design calculations for a correct evaluation of the actual safety margin of a design solution. To incorporate these two sources of uncertainty into reliability analysis, adequate correlation coefficients between geomechanical parameters and construction variables are introduced in Monte Carlo simulation. If only Type 1 uncertainty was present, these associations would be complete; however, since subjectivity and unmanageable variability do exist, these correlations are incomplete.

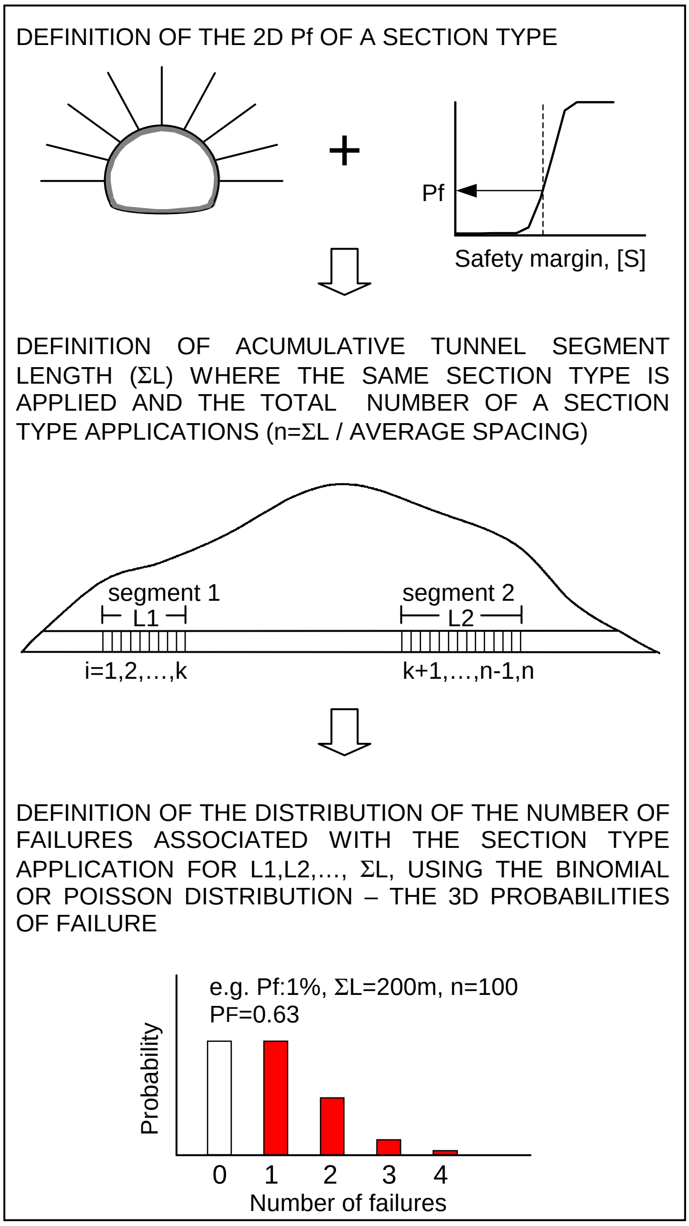


Figure 4. Scheme for obtaining the 3D probability of failure, PF, of a support-section type along the tunnel from the section type 2D probability of failure, Pf.

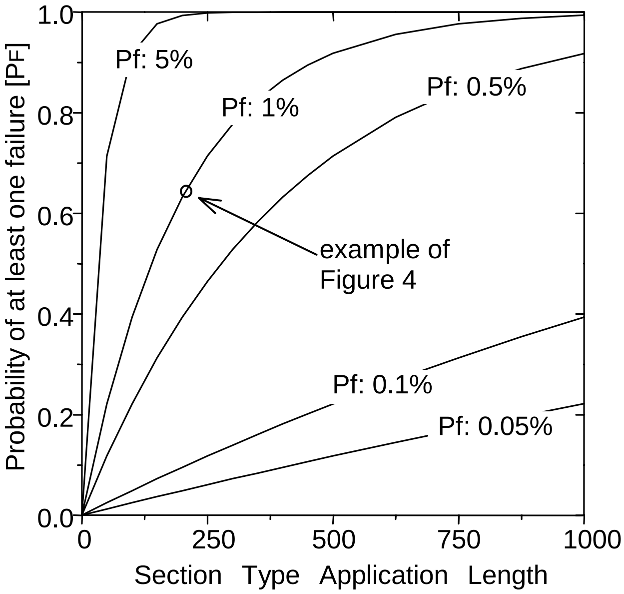


Figure 5. Calculated probability of having at least one failure for a given accumulative length of application of a section type at different specified levels of its 2D failure probability, using a Poisson distribution.

Incorporating into the analysis correlations between capacity and demand variables, is not only in accordance with the creditable recommendations in the literature on structural reliability analysis (e.g. Harr 1987), but also to the fundamental principle that different demands, D, should be satisfied, with a reasonable correlation, by different design solutions with adequate capacity, C. This means that in an optimized design, the two pdfs of C and D may have a certain area of overlapping without increasing the probability of failure. In practical terms, this criterion prevents overdimensioning of the support, and guarantees a more balanced distribution of safety margin for each typical section for the anticipated range of conditions. The variability associated with those expected conditions is to be managed during construction with precise rules for associating geomechanical conditions with support type and density.

6 CONCLUSIONS

The theoretical and practical studies presented in the previous sections of this paper allow for the following conclusions to be made:

- the conventional factor of safety, calculated deterministically as the ratio of a chosen capacity and an anticipated demand, is often found to be inadequate since uncertainty dominates design in rock engineering;
- the safety margin, as calculated from Capacity – Demand model, is a logical replacement of the conventional factor of safety;
- various probabilistic techniques are now readily available to assist designers in moving from deterministic to probabilistic or reliability-based design of a tunnel support system; the example applications presented in section 4 have demonstrated that the proposed reliability evaluation procedure in Figure 1, despite some limitations associated with the convergence-confinement method, is quite effective and efficient for determining the reliability of a support section which is also important from a legal and contractual point of view; the proposed procedure can be readily adapted not only to other solutions of the convergence - confinement method but also to other areas of geotechnical design such as the analysis of the reliability of a foundation system; and
- the differentiation between the 2D and the 3D probability of failure of a section type as presented in section 5 is of practical importance, since it provides the designer with an effective tool for communicating the actual risks involved in the design to the owner and the contractor.

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