Reliability analysis of tunnel-support systems

Analyse de fiabilité des systèmes de soutènement des tunnels

Zuverlässigkeitsanalyse von Tunnelausbausysteme

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ABSTRACT: Common practice in tunnel design considers a deterministic approach for dimensioning primary support and final lining. Generally the design is based on some parameters statistics (for example the mean) and a fixed factor of safety is required to take into account different sources of uncertainty. However, this index is not sufficient to quantify the reliability of a support structure, and can be easily shown that to the same factor of safety could correspond different values of probability of failure. In this paper, the basic concepts of the probabilistic design are shortly reviewed and some practical applications in tunneling are described as developed by the authors, showing the advantages obtained with respect to implementation of the traditional deterministic approach to the same cases.

RÉSUMÉ: La pratique courante pour les études du dimensionnement du soutènement et du revêtement des ouvrages souterrains est basée sur une approche déterministe. Généralement ces études sont basées sur quelques valeurs statistiques telles que la moyenne de certains paramètres, un facteur de sécurité fixe est exigé pour prendre en considération l'incertitude liée à ces valeurs. Toutefois, cela n'est pas suffisant pour quantifier la fiabilité de la structure. Il peut être facilement démontré qu'il existe pour le même facteur de sécurité différentes probabilités de rupture. Cet article expose brièvement les concepts de base des méthodes probabilistes appliquées au dimensionnement des ouvrages et présente des applications pratiques aux ouvrages souterrains, conduites par les auteurs, montrant les avantages obtenus par rapport à l'approche déterministe traditionelle.

ZUSAMMENFASSUNG: In der üblichen Entwurfspraxis werden für die Berechnung von Tunnelausbauten und -innenschalen deterministische Methoden angewandt. Die Bestimmung des Sicherheitsfaktors auf der Basis von einigen statistischen Parametern (z.B. dem Durchschnittswert) reicht micht aus, um die Zuverlässigkeit von Tunelausbausysteme zu quantifizieren. Dem Selben Sicherheitsfaktor Können verschiedene Bruchwarscheinlichkeiten entsprechen.
In diesem Bericht werden die Basiskonzepte lines probabilistischen Entwurfes erneut dargestellt und einige Auwendeungsbrispiels berichtet. Sie zeigen die Vorteile des probabilistischen uin Vergeich zu dem deterministischen Entwurf.

1.INTRODUCTION

The tunnel design problem is basically a decision problem and the decisions must be made usually under conditions of uncertainty due to the many non-decision variables, such as the ground conditions, which are not subject to control by the designer. Therefore, the decisions always involve a certain degree of risk (Xu et al., 1996; Einstein et al., 1998).

Although decisions under uncertainty are routinely dealt with in other fields of engineering and science, they are only occasionally addressed in tunneling due to the complexity of the problems and a lack of suitable and effective tools. In fact, the current, common practice to tunnel design, particularly the dimensioning of primary support and final lining, is to use deterministic approaches incorporating engineering judgement and established design principles.

The single input values are generally representing the "best-estimates" of the parameters and cannot account for either the inherent variability or the uncertainty in the parameters, and the factor of safety, commonly defined as the ratio between the available capacity of the designed support and the demand for support of the excavation, is often found to be inadequate for quantifying the reliability of the system. For the latter case, it can be easily demonstrated that two different tunnel sections having the same factor of safety may have quite different probability of failure (see also Figure 1).

The current, unsatisfactory situation can be improved through application of probabilistic approaches to design as it practised in the field of structural design, incorporating explicitly the various sources of uncertainty and variability in design analysis.

Following a brief review of the basic concepts of probabilistic design, two particular methods, namely the Monte Carlo simulation technique and the Point Estimate method, will be presented and applied to assess the reliability of the primary support and the final lining of a railway and highway tunnels, respectively.

The results obtained from the probabilistic analyses are compared with those obtained from deterministic analyses to demonstrate both the need and advantage of moving towards reliability-based design in tunneling.

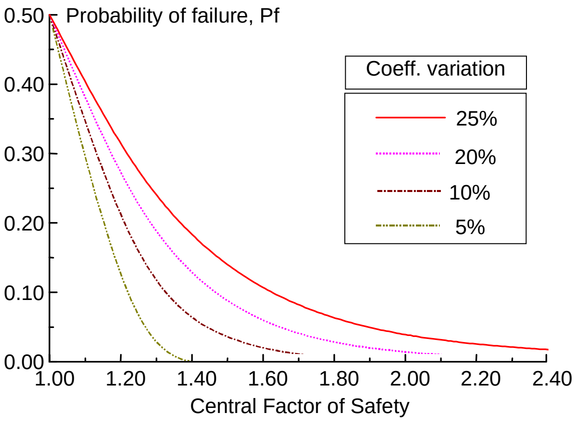

Figure 1. Variation of probability of failure with respect to the central factor of safety, defined as expected capacity over expected demand, for different coefficients of variation (Bieniawski et al., 1994)

2.RELIABILITY-BASED DESIGN

A probabilistic study allows uncertainty related to a parameter (or a random variable) to be integrated in the analysis through the use of probability density functions (pdf). Various sources of uncertainty can be compared, analyzed and combined using a probabilistic procedure. For a given level of uncertainty in the problem, the implied level of reliability can also be quantified, thus allowing for comparison of the safety (reliability) of alternative designs (Tang, 1993).

A practical way to assess the reliability of a design solution is to consider the safety margin (*S*), which is defined by the difference between capacity (*C*) and demand (*D*). Inadequacy of a design is considered within the negative portion of the safety margin distribution:

$$Pf = P[(C\text{-}D)\leq 0] = P[S\leq 0]. \quad (1)$$

Another measure of a design adequacy is the reliability index, β, defined as the inverse of the coefficient of variation of *S* (mean $\mu(S)$ over the standard deviation $\sigma(S)$):

$$\beta = \mu(S)/\sigma(S). \quad (2)$$

In general, any reliability-based analyses shall consist of the following steps:

1. Definition of the empirical, analytical or numerical model that is suitable for the ground conditions-structure interface.
2. Definition of the character of the input variables, deterministic or probabilistic (stochastic).
3. Fitting the appropriate pdf to the collected data and/or assignment of an adequate pdf to the stochastic variables.
4. Incorporation of the different sources of uncertainty in the design analysis methods. There are mainly three approaches for doing so:
   - Monte Carlo (MC) simulation (Metropolis and Ulam, 1949) where repeated samples are taken from actual or estimated pdf of the variables which enter in a function *f* (e.g. support capacity) until the distribution of this function is defined with acceptable precision.
   - Taylor series (First Order Second Moment, FOSM method), where Taylor's formula is used for expanding a function *f* about the average value $\bar{x}$ up to the quadratic term.
   - The Point Estimate Method, PEM (Rosenblueth, 1975), where only two values for each input variable are used to calculate the basic moments of a function f.
5. Reliability analysis of the design solution and investigation of its sensitivity to the input variates.
6. Optimization of the construction practice to maximize the reliability of the design solution selected.

In the following sections example applications of reliability analysis of primary support (3) and final lining (4) using MC and PEM, are respectively described.

3.DETERMINING THE RELIABILITY OF A PRIMARY SUPPORT

3.1 Description of the method

The analyses have been realized using the well-known convergence-confinement method (CCM), following this schematic procedure (see Figure 2). In particular, the approach adopted consists of the following phases:

- evaluation of the variability of the geomechanical and construction parameters involved, and definition of their pdf;
- determination of the correlation matrix of the parameters;
- application of the MC method, using the Latin-Hypercube sampling scheme and rank correlation;
- employment of CCM using as input each time a set of sampled parameters values;
- application of the bisection method for evaluating the intersection point between the support and ground-reaction curves; this point represents the equilibrium condition (p_{ireq}, u_{eq}) and then in terms of internal pressure the "Demand" (D) of the system; an insight of the employed method is given in 3.2;
- evaluation of the safety margin as the difference between the available Capacity (C) of the support and the Demand (D) for support by the excavation;
- repetition of the above process for a statistically sufficient number of iterations;
- analysis of the safety margin distribution and calculation of the reliability index, β and probability of failure, *Pf*.

3.2 An example-application

The example refers to a 100m^2 railway tunnel of about 7000m of length in South America, excavated in complex schistose rocks. The original design included shotcrete and bolts as the primary support system for fair rock mass conditions (class III of RMR, Bieniawski 1984) under an overburden ranging between 150 to 200m. In order to evaluate possible optimizations of the design, a reliability analysis was performed taking in account the variability and uncertainty of the geomechanical and construction parameters. The previously described procedure was applied using the parameters distributions given in Table 1.

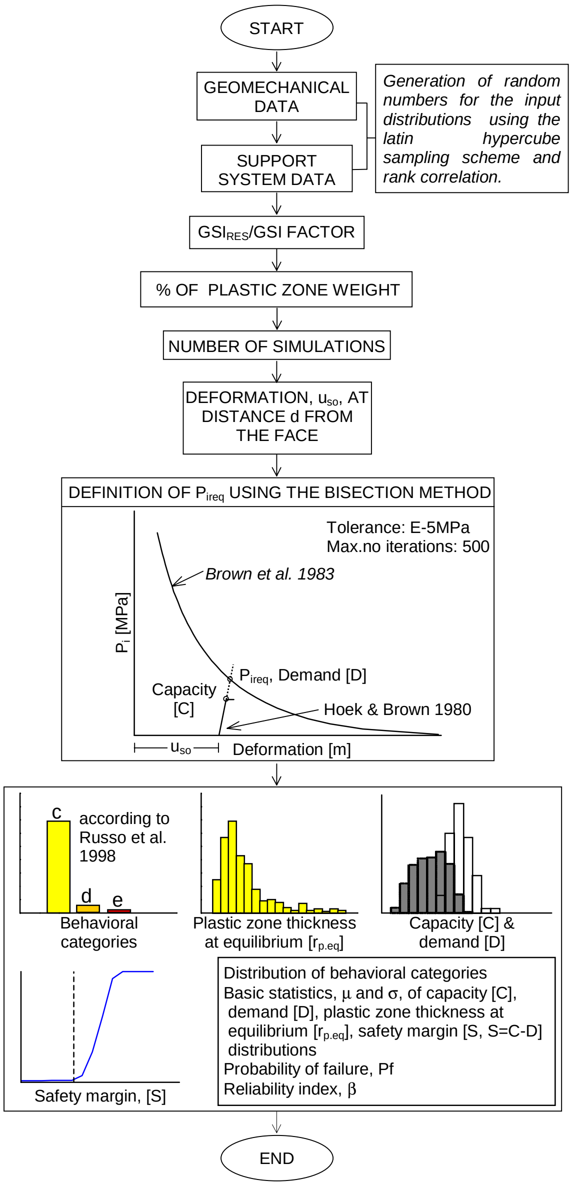

Figure 2. Scheme for reliability evaluation of a primary support system using Monte Carlo simulation and the convergence-confinement method

Table 1. Statistical distributions and correlations for the geological and construction parameters

Input variables	Distributions
Intact rock strength, σ_c, MPa	[10,20,50]
Geological Strength Index, *GSI*	[10,20,50]
Shotcrete thickness, $t_{shot.}$, m	original: [0.05,0.10,0.15], alternative: [0.15,0.20,0.25]
Shotcrete strength, $\sigma_{c.shot.}$, MPa	[10,12,15]
Steel set spacing, *S*, m	alternative: [1.00,1.25,1.50]
Longitudinal & circumferential bolt spacing, s_c & s_l, m	original: [0.90,1.00,1.10]
Bolt capacity, T_{bf}, MN	original: [0.10,0.15,0.20]

Correlation matrix	σ_c	*GSI*
σ_c		0.75
GSI	0.75	
$t_{shot.}$	-0.50	-0.75
s_c & s_l	0.50	0.75
S	0.50	0.75

*All pdfs are of triangular type and defined by [min,mode,max].

The basic assumptions for CCM analysis conducted were:

- The closed-form solution of Brown et al. (1983), for elastic-brittle-plastic material was used to model the response the ground;
- the gravitational load of the plastic zone above the crown (Hoek and Brown,1980) was taken into account;
- the Hoek and Brown criterion parameters, *m* and *s*, were derived from the Geological Strength Index (*GSI*, Hoek, 1994; Hoek et al., 1995);
- residual values (m_r,s_r) were calculated considering a reduced *GSI* value ($GSI_{RES}=0.36GSI$, Russo et al., 1998);
- for the dilatancy parameter, an average value was considered between the hypothesis of deformation at constant volume and the maximum dilatancy calculated according the theory of plasticity (associated flow rule);
- the stiffness and capacity of the supports were calculated according to the equations proposed by Hoek and Brown (1980);
- the critical parameter of deformation at the time of support installation (u_{so}) is calculated according to the principle of similitude of Corbetta and Nguyen Minh (Panet, 1995).

As it is shown in Figure 3, the analysis revealed that, in spite of a central factor of safety (CFS) of 1.2, the original design solution was associated with an unacceptable probability of failure of about 19%. In the same figure the safety margin cumulative probability curve of the proposed alternative design solution, which consisted of steel ribs and shotcrete, is also shown. The latter design solution has a CFS equal to 1.6 and a practically zero probability of support capacity being exceeded by the required pressure for ground-support equilibrium. The analyses clearly demonstrate the inadequacy of the factor of safety in completely depicting the actual reliability of a support system.

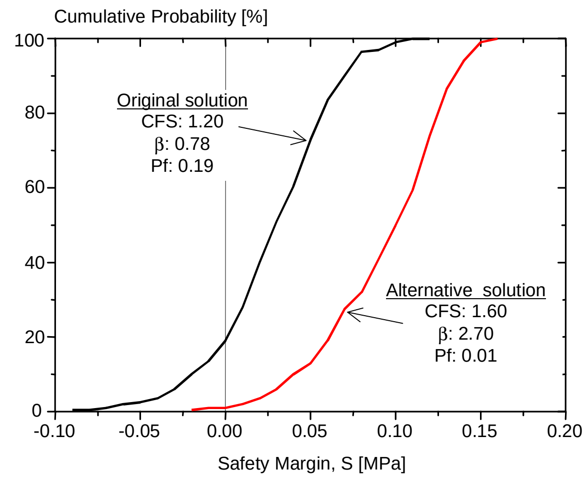

Figure 3. Cumulative distributions of safety margin for original (shotcrete and rockbolts) and alternative (shotcrete and steel sets) primary support solutions for a railway tunnel in South America

4.DETERMINING THE RELIABILITY OF A FINAL LINING

4.1 The analysis procedure

In the previous sections a reliability analysis method combining MC simulation and a simple analytical technique was presented. Needless to say that a more rigorous approach, especially in the phase of detailed design, requires the employment of numerical methods. However, due to the complexity of the algorithms and calculations involved some practical problems arise in using MC and FOSM methods. In this case, a more feasible and suitable way to run a reliability analysis is to use the PEM. This method allows for the calculation of the basic statistics of an unknown variable which is a function of a set of variables whose basic statistics are known.

In particular, for two random variables, Rosenblueth considered the pdf of a symmetric distribution to be analogous to a distributed vertical load acting on a rigid plate supported at four points which are defined by the combinations of the two point estimates of the input variables, x_1 and x_2 (Table 2). The reactions, p_{++}, p_{+-}, p_{-+}, p_{--}, are functions of the correlation coefficient, ρ, between x_1 and x_2, and defined by equations 3 and 4 for the case of symmetric x_1 and x_2 pdfs.

Table 2. PEM coefficients for a bivariate functional relationship

$y=f(x_1,x_2)$	Point-estimates of x_1 and x_2		PEM coefficients
y_{++}	$\mu(x_1)+\sigma(x_1)$	$\mu(x_2)+\sigma(x_2)$	p_{++}
y_{+-}	$\mu(x_1)+\sigma(x_1)$	$\mu(x_2)-\sigma(x_2)$	p_{+-}
y_{-+}	$\mu(x_1)-\sigma(x_1)$	$\mu(x_2)+\sigma(x_2)$	p_{-+}
y_{--}	$\mu(x_1)-\sigma(x_1)$	$\mu(x_2)-\sigma(x_2)$	p_{--}

where: $p_{++}=p_{--}=(1+\rho)/4$ (3), $p_{+-}=p_{-+}=(1-\rho)/4$ (4).

The *M*th moments of *y* are given by:

$$E(y) = p_{++}\, y_{++}^M + p_{+-}\, y_{+-}^M + p_{-+}\, y_{-+}^M + p_{--}\, y_{--}^M \,. \quad (5)$$

To evaluate the reliability of a final concrete lining the PEM is applied in combination with the Statically-Indeterminate Reaction Method (SIRM), that models the behavior of a lining system under applied forces representing the surrounding ground. The structure is discretized according to the finite element method into groups of different elements, connected through rigid nodes and hinges. The analysis refers initially to the neutral axis, verifying later in each section the corresponding stresses (SI and SE) acting in the borderlines (intrados and extrados of the lining).

4.2 An example application

The example refers to the study of dimensioning the tunnel lining of a new highway in Brazil. In the examined zone the overburden is about 150m and the tunnel is expected to be excavated in micashists with the following geomechanical parameters: *GSI* =30±2; σ_c=10±2MPa; m_i= 5±1, where m_i is the Hoek-Brwon criterion parameter for intact rock.

The basic statistics and the corresponding best-fitted distributions of these parameters for input to the reliability analysis using PEM are summarized in Table 3, where the values of cohesive strength, *c*, friction angle ϕ, deformation modulus *Ed* are derived according to the equations suggested by Hoek and Brown (1997). The active load, *Pv*, and the subgrade reaction *k*, derived respectively with Terzaghi and Galerkin formulations (AFTES,1993; see note) are also given in Table 3.

Table 3. Probabilistic representation of main parameters (after 1000 MC simulations)

Parameter	μ	σ	Best-fitted pdf	x^2	p-level*
c (MPa)	0.24	0.02	Weibull 0.18,0.07,3.27	26	0.50
ϕ (°)	21	1	Weibull 19,2,3.48	23	0.70
Ed (GPa)	1.0	0.1	Beta 96.1,62.8,1.7	34	0.16
Pv (MPa)	0.214	0.058	Normal 0.214, 0.058	28	0.44
k (MN/m^3)	164	11	Normal 164, 11	27	0.51

*maximum significance level at which the adopted pdf is suitable to represent the statistical distribution of the data (Benjamin and Cornell,1970; Kulatilake,1993). $Pv = (\gamma B\text{-}2c)/2\tan\varphi$ (6); $k = E_d/(R(1+\nu))$ (7) where γ: unit weight of the rock mass; *B*: width of loading ground arch; *R*: the equivalent radius of the tunnel (5m); $E_d = ((\sigma_c /100)^{0.5})*10^{((GSI-10)/40)}$ (8) and ν is Poisson's ratio.

On the basis of this data, analyses were performed with SIRM considering as input variables *Pv* and *k* only. In particular, according to the PEM the combinations reported in Table 4 were considered to evaluate the stress *S* in the 40cm thick concrete lining. A negative correlation, ρ=-0.75, between *k* and *Pv* was introduced in the analyses.

Table 4. Parametrical combinations for PEM analysis

Variable	Analysis: 1	Analysis: 2	Analysis: 3	Analysis: 4
k (MN/m^3)	(164+11)	(164+11)	(164-11)	(164-11)
Pv (MPa)	(0.214+0.058)	(0.214-0.058)	(0.214+0.058)	(0.214-0.058)
S	S_{++}	S_{+-}	S_{-+}	S_{--}

Figure 4 shows the load configuration on the lining and the position of some representative structural nodes. For each of these nodes the expected value E(*S*) and the standard deviation σ(*S*) of the stress were calculated according to the proposed procedure, to derive the pdf of the stress. The resulting distributions were analyzed with respect to the compressive, f_{ck}, and tensile strength, f_{ctk}, of the concrete as well as to the reduced values, f_{cd} and f_{ctd} respectively, of these two indices as suggested by CNR(1980) for limit state analysis. This reduction was considered necessary to account for the actual variation in the concrete strength. The actual pdf of concrete strengths should be used instead of this reduction when available. In Table 5 and Figure 5 the principal results are summarized. Nodes 1 and 6, in the invert, are overloaded. An unacceptable probability of exceeding the reference strength values is associated with these nodes. In particular, the probabilities of failure (ultimate strength exceeded) are 6-11% and 72-81% for the compressive and for the tensile stress analysis, respectively. The need for an adequate steel reinforcement of the invert is consequently evident.

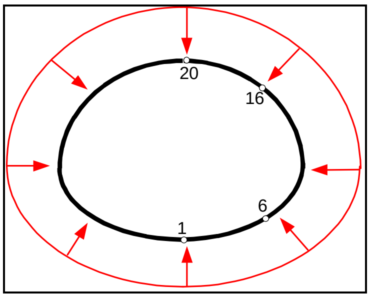

Figure 4. Configuration of the load on the lining (numbers indicate the nodes for which PEM analysis was made)

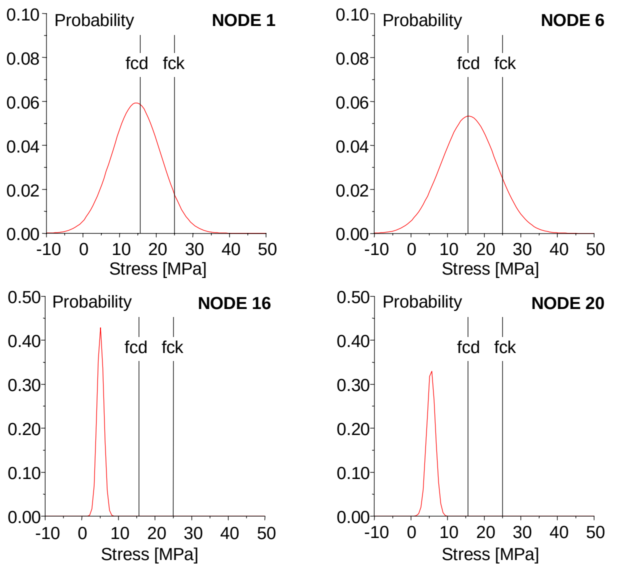

Figure 5. Distribution of compressive stresses as resulted from the PEM analysis for the nodes defined in Figure 4 (positive values denote compression)

Table 5. Selected results from PEM-SIRM analysis

Node	Statistical data E(SI) [MPa]	σ(SI) [MPa]	(Sdet.) SI,max [MPa]	Statistical data E(SE) [MPa]	σ(SE) [MPa]	(Sdet.) SE,max [MPa]
1	-3.9	3.4	-3.9	14.6	6.7	14.4
6	15.9	7.5	16.2	-5.0	3.9	-5.4
14	5.1	0.9	5.3	4.5	2.1	4.4
20	3.9	1.8	3.9	5.5	1.2	5.6

Node	Check for compress. stress Smax	P(>fcd)	P(>fck)	Check for tensile stress Smax	P(>fctd)	P(>fctk)
1	SE	4.4E-01	6.2E-02	SI	8.1E-01	7.5E-01
6	SI	5.2E-01	1.1E-01	SE	7.5E-01	7.2E-01
14	SI	<1.0E-10	<1.0E-10	-	-	-
20	SE	<1.0E-10	<1.0E-10	-	-	-

fck = 25MPa; fcd = fck/1.6; fctk=1.6MPa; fctd=fctk/1.6; P(>f...) = cumulative probability of exceeding the reference strength. Negative, positive values = tensile, compressive stresses.

In Table 5 are also reported for comparison purposes the results obtained from the deterministic analysis (Sdet.), using the mean values of *Pv* and *k*. A good agreement is observed between the expected value from the PEM analysis and the deterministic one. Nevertheless, the limitation of the latter is evident; for example, the deterministic analysis shows that for node 6 only the “reduced” strength is exceeded, while the reliability analysis indicates that P(>fck) is 11%.

5.FURTHER DEVELOPMENT: ASSESSING DIFFERENT TYPES OF UNCERTANTY IN DESIGN

In tunnel design, more than in other fields of geoengineering, the geomechanical model is generally extrapolated from a limited amount of data and the real state of the ground is only partially known. As of this, the design should consider reasonable geomechanical scenarios, adequate solutions for these expected conditions and rules for applying the solutions during the excavation. At this point it becomes important to classify correctly the geomechanical context, identify the appropriate support design choices and then apply the selected support scheme to stabilize the excavation. In this manner, the design process involves two types of uncertainty, the following:

– Type1 which represents uncertainty related to the inherent variability of a parameter
– Type2 which depicts the uncertainty in evaluating the real state of a parameter.

For example, on the basis of the past experience, in a relatively homogeneous zone GSI is expected to vary between 40 to 60 according to a pdf. This spatial variation corresponds to type1 uncertainty. The design solution, by predefined rules, shall manage this variability. However, the recorded data can be affected by type 2 uncertainty, due to subjectivity and/or objective difficulty in evaluating the state of this geomechanical index at the tunnel face.

The same concept can be applied to the construction parameters: here, type1 variability is induced and is managed during construction, while type2 is random and cannot be governed. For instance, for a certain section the design value of shotcrete thickness is 15±5cm, while the actual value is selected at the face according to a predefined design rule. Nevertheless, a practical tolerance of ±1cm shall also be considered in the design solution although it cannot be managed.

To take into account the different uncertainties in the reliability analysis, the following scheme is proposed for geomechanical and construction parameters:

– Type1 determines the basic pdf
– Type2 introduces variability in the statistics of these pdfs.

In Table 6, the general approach applied for quantifying type1 & type2 uncertainties is given, while Table 7 provides a practical application of the approach showing the input design parameter for a specific tunnel zone. In this zone a particular section type is designed, considering a variable intensity of the stabilization measures in function of the actual geomechanical conditions. In order to correctly simulate this concept of flexible design, where the support is modulated according to the real necessity (encountered geomechanical conditions), the variables entering in the Capacity and Demand functions are considered correlated (e.g. ρ = 0.75). This means that the resulting demand and capacity pdfs can overlap without the area of overlapping to correspond to the cumulative probability of capacity being exceeded by demand. This important concept prevents over-dimensioning of the support.

Table 6. Suggested approach for quantifying uncertainties

Uncertainty	Geomechanical parameters	Construction variables
Type1	pdf	Induced (design)
Type2	- 99% confidence limits for μ and σ* and/or - expert judgement**	Expert judgement and/or bibliographical data

*when sufficient data series are available, the simplified assumption is adopted that type2 uncertainty is assigned to the statistics of the pdf considering the 99% confidence limits of these values; ** in other cases, expert judgement is used to transfer the uncertainty associated with the evaluation of a parameter state to the statistics of the pdf.

Table 7. An example of definition of different pdfs to describe uncertainty Type1 and Type2 and their combined pdf representation

Parameter	Type1 uncertainty	Type2 uncertainty	Combined (Type1&2)
GSI	Gamma: (137,0.36)	$\mu = \pm 2$	Gamma: (125-147, 0.38-0.34)
σ_c (MPa)	Gamma: (3.8,12.8)	$\mu = \pm 5$	Gamma: (3.1-4.7, 14.2-11.6)
m_i	Triangular: (10-12-14)	±1	Triangular: (9-11, 11-13, 13-15)
Shotcrete (3day strength, MPa)		Triangular: (16,20,24)	Triangular: (16,20,24)
Shotcrete thickness (cm)	Triangular: (10,15,20)	±1	Triangular: (9-11, 14-16, 19-21)
Steel ribs spacing (m)	Triangular: (1.00,1.25,1.50)		Triangular: (1.00,1.25,1.50)

Example: *GSI*- Type1: a Gamma (shape:137, scale:0.6) distribution is the best-fitted curve for the collected data in a geomechanical zone similar to the one to be excavated. Type2: is assigned considering a possible variation of the mean of ±2. Combined (1&2): the resulting Gamma distribution is characterized by a variability of the parameters given from type2 uncertainty. To take into account this variability, during the probabilistic simulation a dynamic Latin Hypercube sampling is implemented .

6.CONCLUSION

The principal conclusions of this paper are:

- the conventional, deterministically-defined factor of safety is generally inadequate for depicting the actual reliability of any tunnel support system;
- to quantify the reliability of a tunnel support, three additional, probabilistically-defined indices, namely the Safety Margin, the Reliability Index and the Probability of Failure may be introduced;
- depending on the design phase, consequently the level of accuracy required, either analytical or numerical models can be used to simulate the ground and tunnel structure interaction; in the former case it is most convenient to use the Monte Carlo (MC) technique to simulate the various sources of uncertainty and variability in the design-input parameters, while in the latter case it is best to apply the Point Estimate Method (PEM) to account for the uncertainties;
- using the probabilistic approach, the reliability of the support system is explicitly calculated and thus the risks involved in tunnel design are basically known. On the same bases, alternative design solutions can be easily and effectively compared.

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